Unified International Mathematics Olympiad

UIMO SAMPLE QUESTIONS

CLASS - 12

MATHEMATICS - 1

01. **If $1 < x < \sqrt{2}$, then the number of solutions of the equation $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$ is:**

(A) 0 (B) 1 (C) 2 (D) 3

02. **$dx + dy = (x + y)(dx - dy) \Rightarrow \log(x + y) =$ ________**

(A) $x + y + c$ (B) $x + 2y + c$ (C) $x - y + c$ (D) $2x + y + c$

03. **If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then find the value of $\begin{vmatrix} 2a_1+3b_1+4c_1 & b_1 & c_1 \\ 2a_2+3b_2+4c_2 & b_2 & c_2 \\ 2a_3+3b_3+4c_3 & b_3 & c_3 \end{vmatrix}$**

(A) Δ (B) 2Δ (C) $\frac{1}{2}\Delta$ (D) $(2\times3\times4)\Delta$

04. **Evaluate: $\int \frac{dx}{(2ax+x^2)^{\frac{3}{2}}}$**

(A) $\frac{-(x+a)}{\sqrt{2ax+x^2}}+C$ (B) $\frac{-1}{a}\frac{x+a}{\sqrt{2ax+x^2}}+C$ (C) $\frac{-1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}}+C$ (D) $\frac{-1}{a^3}\frac{x+a}{\sqrt{2ax+x^2}}+C$

05. **Find the area enclosed between the graph of $y = \cos x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and the X – axis.**

(A) 2 (B) 1 (C) π (D) $\frac{\pi}{2}$

MATHEMATICS - 2

01. **Let $R = \{(x, y) : x, y \in \mathbb{R}, x^2 + y^2 \le 25\}$ $R' = \{(x, y) : x, y \in \mathbb{R}, y \ge \frac{4}{9}x^2\}$ then**

(A) dom $R \cap R' = [-3, 3]$ (B) Range $R \supset R' = [0, 4]$

(C) Range $R \cap R' = [0, 5]$ (D) $R \cap R'$ defines a function

02. **If x, y, z are not all zero and if $ax + by + cz = 0$**

$bx + cy + az = 0$ then $x : y : z =$

$cx + ay + bz = 0$

(A) $1 : 1 : 1$ (B) $1 : \omega^2 : \omega$ (C) $1 : \omega : \omega^2$ (D) $a : b : c$

03. **Let h(x) = min{x, x^2}, for every real number x. Then**

(A) h is continuous for all x

(B) h is differentiable for all x

(C) h'(x) = 1, for all x > 1

(D) h is not differentiable at two values of x

04. **Let $f : \mathbb{R} - \{0\} \to \mathbb{R} - [-1, 1]$, defined by $f(x) = \dfrac{e^{x^3} + e^{-x^3}}{e^{x^3} - e^{-x^3}}$, then f is**

(A) a one-one function

(B) an increasing function

(C) a decreasing function

(D) onto function

05. **If $\int \log\left(\sqrt{1-x} + \sqrt{1+x}\right)dx = xf(x) + Ax + B\sin^{-1}x + c$, then**

(A) $B = -\dfrac{1}{2}$

(B) $A = -\dfrac{1}{2}$

(C) $B = \dfrac{2}{3}$

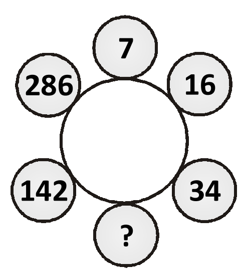
(D) $f(x) = \log\left(\sqrt{1-x} + \sqrt{1+x}\right)$

REASONING

01. **Choose the odd-one out.**

(A) 2731 (B) 1357 (C) 2571 (D) 2357

02. **Find the missing term.**



(A) 72 (B) 70 (C) 68 (D) 66

03. **Which term will replace the question mark in the series?**

ABD, DGK, HMS, MTB, SBL, ?

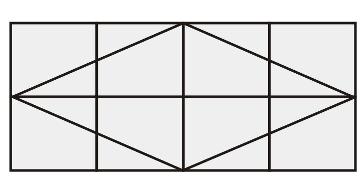
(A) ZKW

(B) ZKU

(C) ZAB

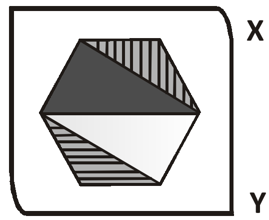
(D) XKW

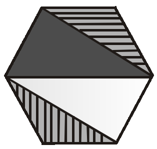
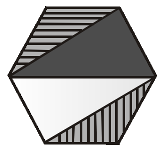
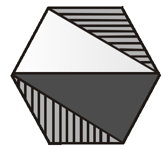
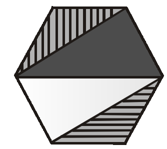
04. **How many triangles are there in the following figure ?**



(A) 22 (B) 18 (C) 20 (D) 24

05. Find mirror image if XY denotes the position of mirror.



(A)  (B)  (C)  (D) 

CRITICAL THINKING

01. If P $ Q means P is the brother of Q ; P # Q means P is the mother of Q ; P * Q means P is the daughter of Q in A # B $ C * D, who is the father?

(A) D (B) B

(C) C (D) Data is indaequate

02. 8 persons E, F, G, H, I J, K and L are seated around a square table- two on each side. There are 3 ladies who are not seated next to each other.

J is between L and F.

G is between I and F

H, a lady member is second to the left of J.

F, a male member is seated opposite to E, a lady member.

There is a lady member between F and I.

What is true about J and K ?

(A) J is male, K is female (B) J is female, K is male

(C) Both are female (D) Both are male

03. What is wrong with this argument ?

"You think we need a new regulations to control air pollution? I think we already have too many regulations. Politicians just love to pass new ones, and control us even more than they already do. It is suffocating. We definitely do not need any new regulations!"

(A) The person speaking doesn't care about the environment.

(B) The person speaking has changed the subject.

(C) The person speaking is running for political office.

(D) The person speaking does not understand pollution.

04. **A, B, C, D and E are five men sitting in a line facing to south – while M, N, O, P and Q are five ladies sitting in a second line parallel to the first line and are facing to North. B who is just next to the left of D, is opposite to Q. C and N are diagonally opposite to each other. E is opposite to O who is just next right of M. P who is just to the left of Q, is opposite to D, M is at one end of the line.**

If B shifts to the place of E, E shifts to the place of Q, and Q shifts to the place of B, then who will be the second to the left of the person opposite to O ?

(A) Q (B) P (C) E (D) D

05. **October 1, 2018 is Monday. What day of the week lies on January 1, 2019 ?**

(A) Monday (B) Wednesday (C) Thursday (D) Tuesday

KEY & SOLUTION

MATHEMATICS - 1

01. **(A)** We have, $\tan^{-1}(x-1)+\tan^{-1}(x+1)=\tan^{-1}3x-\tan^{-1}x$

$$\Rightarrow \tan^{-1}\frac{x-1+x+1}{1-(x-1)(x+1)}=\tan^{-1}\frac{3x-x}{1+(3x)(x)}$$

$$\Rightarrow \frac{2x}{2-x^2}=\frac{2x}{1+3x^2} \Rightarrow x+3x^3=2x-x^3$$

$$\Rightarrow 4x^3-x=0$$

$$\Rightarrow x=0 \text{ or } \pm\frac{1}{2}$$

$\because 1<x<\sqrt{2}$, there is no solution.

02. **(C)** $dx+dy=(x+y)(dx-dy)$

$$\Rightarrow \frac{dx+dy}{x+y}=dx-dy$$

$$\Rightarrow \log(x+y)=x-y+c$$

03. **(B)** The required determinant is obtained by the successive operations

$C_1 \to 2C_1$ and $C_1 \to C_1+3C_2+4C_3$

$\therefore$ The value of the determinant is multiplied by 2 (since of the first operation), second operation does not affect the value of the determinant.

04. **(C)** $2ax+x^2=(x+a)^2-a^2$

Put $x+a=a\sec\theta$, so that $dx=a\sec\theta\tan\theta\, d\theta$

$$\therefore I=\int\frac{a\sec\theta\tan\theta}{a^3\tan^3\theta}.d\theta$$

$$=\frac{1}{a^2}\int\frac{\cos\theta}{\sin^2\theta}.d\theta=\frac{-1}{a^2\sin\theta}+C$$

$$=\frac{-1}{a^2}\frac{\sec\theta}{\tan\theta}+C$$

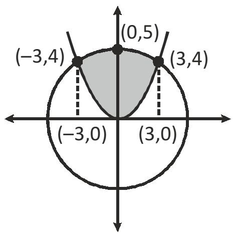
$$=\frac{-1}{a^2}\frac{x+a}{\sqrt{2ax+x^2}}+C$$

05. **(A)** Required area $=\int_{-\pi/2}^{\pi/2}\cos x\,dx$

Note that $\cos x$ is non – negative in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

MATHEMATICS - 2

01. **(A,B,C)** The equation $x^2 + y^2 = 25$ represents a circle with centre (0, 0) and radius 5 and the equation $y = \frac{4}{9}x^2$ represents a parabola with vertex (0, 0) and focus (0, 5)



Hence, from the figure, we have

$R \cap R' = \{(x, y): -3 \leq x \leq 3, 0 \leq y \leq 5\}$

Thus, dom $R \cap R' = [-3, 3]$ and Range $R \cap R' = [0, 5] \supset [0, 4]$

Since, $(0, 0) \in R \cap R'$ and $(0, 5) \in R \cap R'$

$\therefore$ 0 is related to 0 as well as 5

Hence $R \cap R'$ doesn't defines a function.

02. **(A,B,C)** The system has non-trivial solution if

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$\Rightarrow \Delta = 3abc - a^3 - b^3 - c^3 = 0$

$\Rightarrow -\frac{1}{2}(a + b + c)\{(b - c)^2 + (c - a)^2 + (a - b)^2\} = 0$

$\Rightarrow a + b + c = 0$ or $b = c = a$

CASE - I :

If $a + b + c = 0$

First two equations can be written as $ax + by - (a + b)z = 0$ and $bx - (a + b)y + az = 0$

$$\Rightarrow \frac{x}{ab-(a+b)^2} = \frac{-y}{a^2+(a+b)} = \frac{z}{-a(a+b)-b^2}$$

$$\Rightarrow \frac{x}{a^2+b^2+ab} = \frac{+y}{a^2+b^2+ab} = \frac{z}{a^2+b^2+ab}$$

$\therefore \boxed{x:y:z=1:1:1}$

CASE - II :

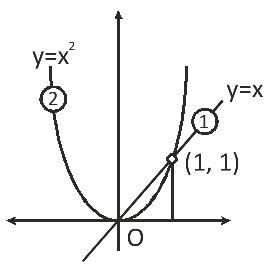
If $a = b = c$, each equation becomes $x + y + z = 0$

$\Rightarrow x = 1, y = \omega, z = \omega^2$ or $x = 1, y = \omega^2, z = \omega$

$\therefore \boxed{x:y:z=1:\omega:\omega^2}$

03. **(A,C,D)** $h(x) = \min(x, x^2) = \begin{cases} x & x<0 \\ x^2 & 0 \le x < 1 \\ x & x \ge 1 \end{cases}$

The graph of min (x, x^2)



It is evident from the graph that the given function is continuous for all x.

Since there are corner points at x = 0 and x = 1, the function is not differentiable at these points.

$\therefore$ the curve does not possess a definite slope at the corner points.

Also, at $x \geq 1$, the function represents the equation of a straight line having slope 1.

$\therefore h'(x) = 1 \ \forall \ x \geq 1$

04. **(A,C,D)** f(x) is discontinuous at x = 0

$-\infty$ ——○—— ∞
0

$$f'(x) = \frac{-12x^2}{\left(e^{x^3} - e^{-x^3}\right)^2}, x \neq 0$$

$\therefore$ f(x) is a decreasing function in $(-\infty, 0)$ and also in $(0, \infty)$ i.e., in $\mathbb{R} - \{0\}$ and hence also a one-one function.

Also, $\lim\limits_{x\to 0\to\infty} f(x) = -1, \lim\limits_{x\to\infty} f(x) = 1$

$\lim\limits_{x\to 0\to 0} f(x) = -\infty, \lim\limits_{x\to 0\to 0} f(x) = \infty$

$\therefore$ Range of f = $(-\infty, -1) \cup (1, \infty) = \mathbb{R} - [-1, 1]$

Since, Range equals Codomain

Hence, f is onto.

05. **(A,B,D)** Let I = $\int \underbrace{\log\left(\sqrt{1-x}+\sqrt{1+x}\right)}_{\text{I}} \underbrace{1}_{\text{II}} dx$

Using ILATE, we have

$$\Rightarrow I = x\log\left(\sqrt{1-x}+\sqrt{1+x}\right) - \int \frac{1}{2}\frac{1}{\sqrt{1-x^2}}\left(\frac{\sqrt{1-x^2}}{x}-\frac{1}{x}\right)x\,dx$$

$$\Rightarrow I = x\log\left(\sqrt{1-x}+\sqrt{1+x}\right) - \frac{1}{2}\int\left(1-\frac{1}{1-x^2}\right)dx$$

$$\Rightarrow I = x\log\left(\sqrt{1-x}+\sqrt{1+x}\right) - \frac{x}{2} + \frac{1}{2}\sin^{-1}x + c$$

$$\therefore f(x) = \log\left(\sqrt{1-x}+\sqrt{1+x}\right), A = -\frac{1}{2}\ \&\ B = \frac{1}{2}$$

REASONING

01. **(C)** Since 2571 is divisible by 3

02. **(B)** 7 × 2 + 2 = 16 70 × 2 + 2 = 142

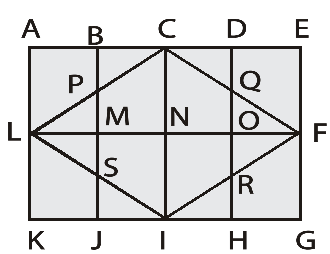
16 × 2 + 2 = 34 142 × 2 + 2 = 286

34 × 2 + 2 = $\boxed{70}$

03. **(A)** ABD, DGK, HMS, MTB, SBL, ZKW

+3 +4 +5 +6 +7

04. **(A)**

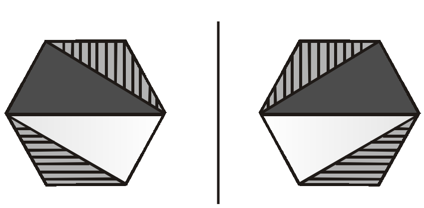


The triangles are:

LPM, LMS, QOF, ROF, JSI, IHR, CDQ, BCP, LCN, CNF, NIF, LNI, LKI, FGI, CEF, ALC, LCF, CFI, LFI, LCI, PSL, QRF

Total = 22 triangles

05. **(D)**

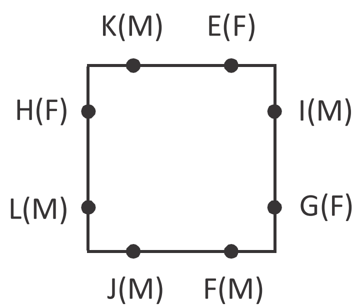


CRITICAL THINKING

01. **(A)** A is the mother of B, B is the brother of C and C is the daughter of D. Hence, D is the father.

A (Parents) D
| |
| |
B – is – Brother – of – C

02. **(D)**

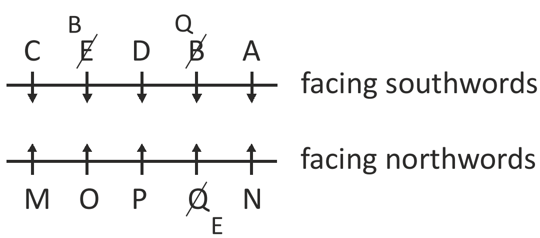


Here M = Male, F = Female

Both are male.

03. **(B)**

04. **(A)**



Opposite to O is B.

So, second left to B is Q.

05. **(D)** On January 1, 2019 it is Tuesday.